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Possible representations of semisimple groups for finite $N=2$ supersymmetric Yang–Mills theories

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Abstract. In this paper we list all possible representations of semisimple groups $G = G_1 \times G_2 \times \ldots \times G_k$ ($k \geq 3$) for finite $N = 2$ supersymmetric Yang-Mills theories, but without consideration of the cases of pseudoreal representations.

Recently a lot of attention has been paid to finite quantum field theories. The first such example is the $N = 4$ supersymmetric Yang-Mills (SYM) theory (Gliotti *et al* 1977, Brink *et al* 1977), which was first proved to be finite in the light-cone gauge formalism (Mandelstam 1983, Brink *et al* 1983). The more general cases of finite quantum field theories are the $N = 2$ SYM theories with $N = 2$ matter multiplets provided their gauge coupling β function vanishes at one loop (Howe *et al* 1983). Attempts have just been made to obtain the sufficient and necessary conditions for a finite $N = 1$ SYM theory (Parkes and West 1984). An $N=2$ SYM theory usually includes an $N=2$ vector multiplet which consists of an $N = 1$ vector multiplet and an $N = 1$ chiral multiplet all in the adjoint representation of gauge group G, and several $N = 2$ matter multiplets each of which consists of an $N = 1$ chiral scalar multiplet in representation R_i and an $N = 1$ chiral scalar multiplet in representation $\bar{R}_i$. Most recently Derendinger *et al* (1984) extended the $N = 2$ matter multiplets to include $N = 1$ chiral scalar multiplets in pseudoreal representations R_{pj} of the gauge group G. The vanishing of β function at one-loop level for simple group G becomes

$$C_2(G) = \sum_i T(R_i) + \tfrac{1}{2} \sum_j T(R_{pj}) \tag{1}$$

where $C_2(G)$ is the value of the quadratic Casimir operator for the adjoint representation of G, and $T(R)$ is the Dynkin index of representation R. For a semisimple group $G = G_1 \times G_2 \times \ldots \times G_k$ where $G_1, G_2, \ldots, G_k$ are simple groups, criterion (1) for a finite $N = 2$ SYM theories can be easily extended to

$$\begin{aligned} C_2(G_1) &= \sum_i T(R_1^{(i)}) \times \dim R_2^{(i)} \times \ldots \times \dim R_k^{(i)} + \tfrac{1}{2} \sum_j T(R_1^j) \times \dim R_2^j \times \ldots \times \dim R_k^j \\ C_2(G_2) &= \sum_i \dim R_1^{(i)} \times T(R_2^{(i)}) \times \ldots \times \dim R_k^{(i)} + \tfrac{1}{2} \sum_j \dim R_1^j \times T(R_2^j) \times \ldots \times \dim R_k^j \\ &\vdots \\ C_2(G_k) &= \sum_i \dim R_1^{(i)} \times \dim R_2^{(i)} \times \ldots \times T(R_k^{(i)}) + \tfrac{1}{2} \sum_j \dim R_1^j \times \dim R_2^j \times \ldots \times T(R_k^j) \end{aligned} \tag{2}$$

where $N = 2$ matter multiplets are in the representations $R^{(i)} = R_1^{(i)} \times R_2^{(i)} \times \ldots \times R_k^{(i)}$ and $\bar{R}^{(i)}$, as well as in the pseudoreal representations $R_{pj} = R_1^j \times R_2^j \times \ldots \times R_k^j$. The

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condition for R_{pj} being pseudoreal is that the number of pseudoreal representations in R^j_l ($l = 1, \ldots, k$) is odd and the other ones are real or singlets. The dimension of representation R is denoted by dim R.

As for a given gauge group G, there are few representations with Dynkin index smaller than C_2(G), there are only finite number of sets of representations satisfying (1). So representations of all classical simple groups satisfying (1) were found (Koh and Rajpoot 1984, Derendinger *et al* 1984) and the candidates for a finite grand unified theory which can accommodate at least three generations of ordinary quarks and leptons were discussed (Dong *et al* 1984, Derendinger *et al* 1984). We have found representations of semisimple groups $G = G_1 \times G_2$ and $G = SU(m_1) \times SU(m_2) \times \ldots \times SU(m_k)$ satisfying (2) when the part of pseudoreal representations in (2) is not considered (Jiang and Zhou 1984a, b). In this paper we will complete the representations of all classical semisimple gauge groups $G = G_1 \times \ldots \times G_k$ ($K \geq 3$) satisfying (2), still not considering pseudoreal representations in (2). The contributions from pseudoreal representations for semisimple groups will be given elsewhere.

We need only consider k (the number of simple groups G_i of G) ≥ 3, and study the cases where at least one subgroup G_i does not belong to SU(m). It is convenient for us to consider Sp($2n$) ($n \geq 2$) and SO($2n+1$) ($n \geq 3$), instead of Sp($2n$) ($n \geq 3$) and SO($2n+1$) ($n \geq 2$). In doing so the complicated cases only appear in the semisimple groups consisting of SU(m) and Sp($2n$). As in the case of $G = SU(m_1) \times \ldots \times SU(m_k)$ (Jiang and Zhou 1984b) we need only consider the irreducible cases where (2) can not be divided into two independent sets of equations. Also we can use a diagram to denote a set of representations of G satisfying (2). We use a dot to denote a subgroup SU(m) and write down m near the dot. A small circle is used to denote the other classical simple group G_i, such as Sp($2n$), SO(n) etc, and G_i is written near the circle. As Sp($2n$) appear many times, we simply write down the number $2n$ near the circle most of the time. In the cases under consideration the representation $R^{(i)} = R^{(i)}_1 \times \ldots \times R^{(i)}_l \times \ldots \times R^{(i)}_t \times \ldots \times R^{(i)}_k$ of $G = G_1 \times \ldots \times G_l \times \ldots \times G_t \times \ldots \times G_k$ can be only non-singlets at most for two subgroups, for example G_l and G_t. There are only some choices in these cases. (1) $G_1 = SU(m_1)$, $G_t = SU(m_t)$ and $R^{(i)}_1$, $R^{(i)}_t$ are fundamental representations $\square$. (2) $G_1 = SU(m)$, $G_t = Sp(2n)$ and $R^{(i)}_1 = \square$, $R^{(i)}_t = R_1$. (3) $G_1 = SU(m)$, $G_t = SO(n)$ and $R^{(i)}_1 = \square$, $R^{(i)}_t = R_f$. (4) $G_1 = Sp(2n)$, $G_t = SO(n)$ and $R^{(i)}_1 = R_1$, $R^{(i)}_t = R_f$. In all these four cases we connect the corresponding dot (or circle) and dot (or circle) by a line. (5) $G_1 = SU(m)$, $G_t = SO(n)$ and $R^{(i)}_1 = \square$, $R^{(i)}_t = R_s$. Here double lines connect the dot of SU(m) and the circle of SO(n). Except for the cases considered above, all $R^{(i)}$ of G can be non-singlets only for one subgroup G_1 of G, and in such cases we write the corresponding representation $R^{(i)}_1$ near the dot (or circle) of G_1. The representations R_1, R_f, R_s etc appearing here are listed in table 2. Hence every set of representations of G satisfying (2) can be expressed by a diagram.

There is only one diagram consisting of circles, which is $\overset{4}{\circ}$——$\overset{6}{\circ}$——$\overset{4}{\circ}$. The other ones have to include dots as well. So we can add circles into the diagrams of $G = SU(m_1) \times \ldots \times SU(m_k)$ to obtain all these diagrams. The exceptional groups E_6, E_7, E_8, F_4 and G_2 can not appear in a diagram when $k \geq 3$. We list all diagrams under consideration in A and B of table 1.

Table 1. All diagrams satisfying (2) without considering pseudoreal representations and with the number of simple subgroups of G greater than two.†

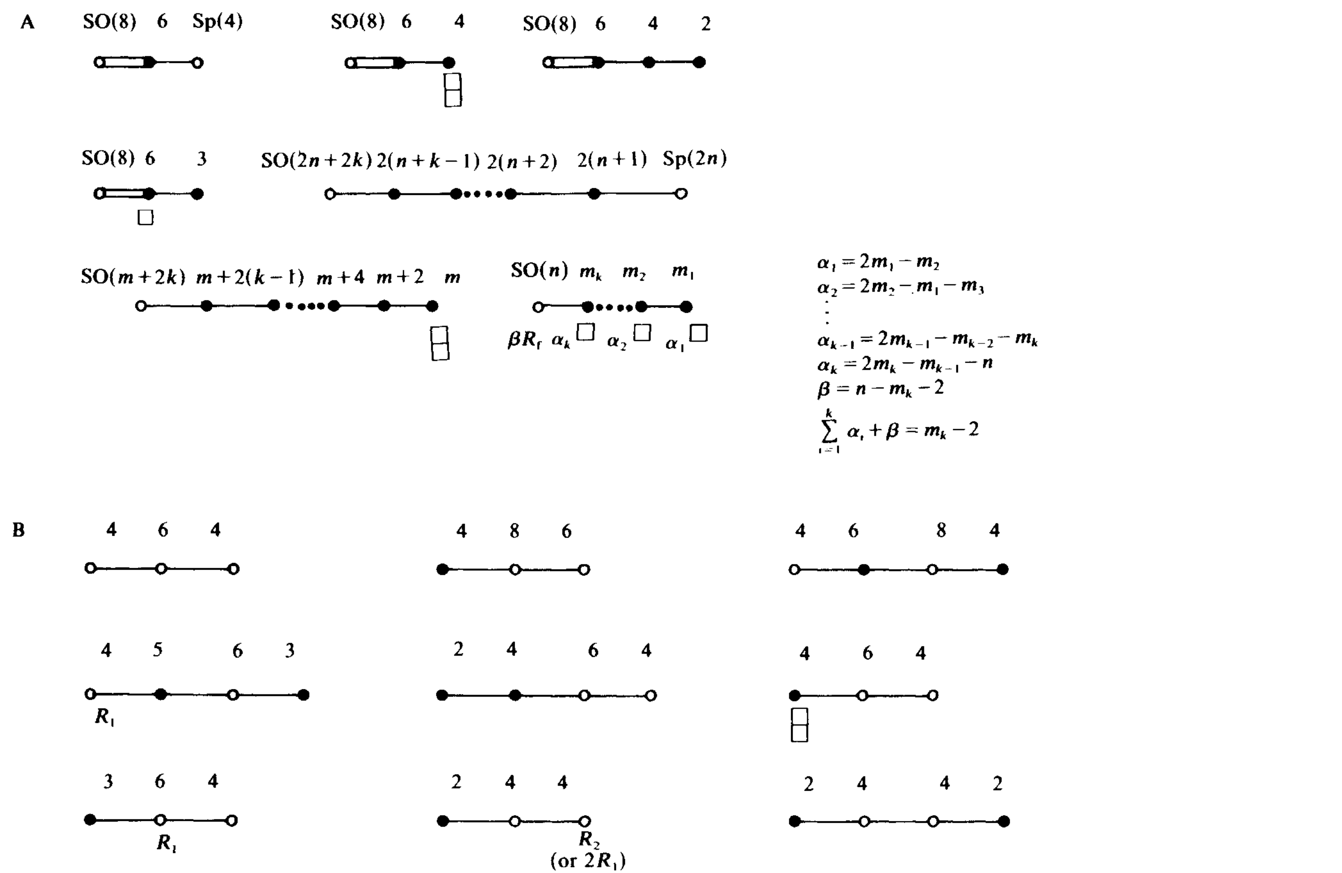
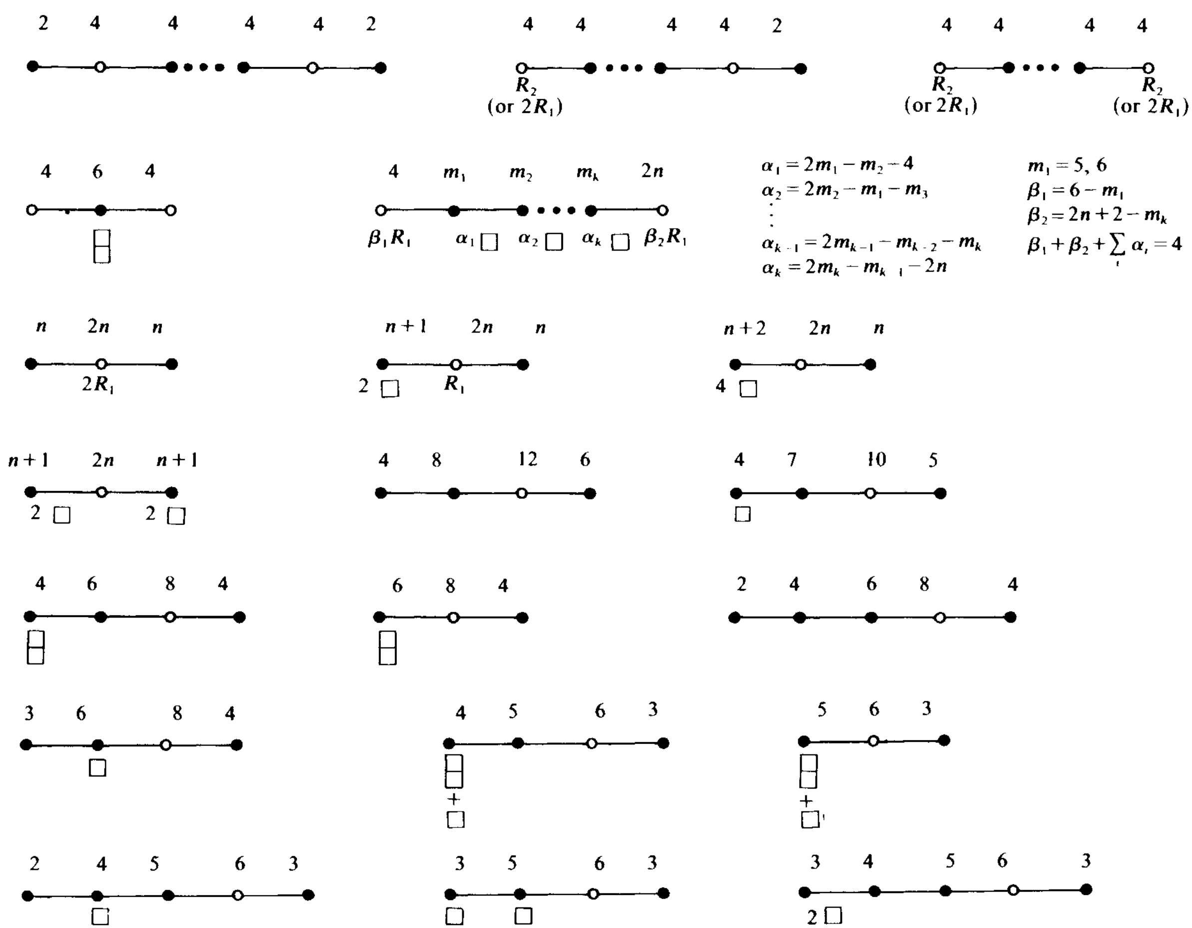


Table 1. (continued)



$\alpha_1 = 2m_1 - m_2 - 4$
$\alpha_2 = 2m_2 - m_1 - m_3$
$\vdots$
$\alpha_{k-1} = 2m_{k-1} - m_{k-2} - m_k$
$\alpha_k = 2m_k - m_{k-1} - 2n$

$m_1 = 5, 6$
$\beta_1 = 6 - m_1$
$\beta_2 = 2n + 2 - m_k$
$\beta_1 + \beta_2 + \sum_i \alpha_i = 4$

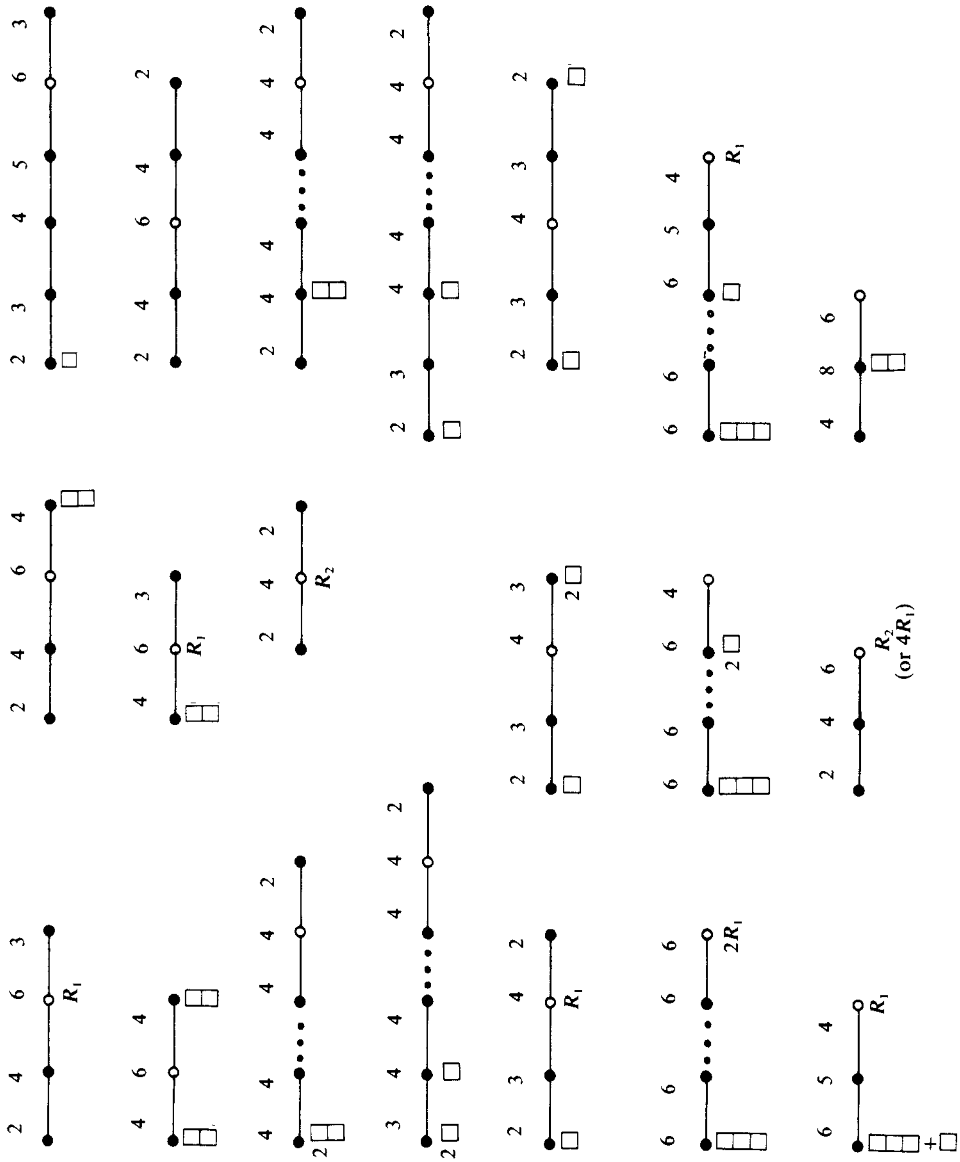
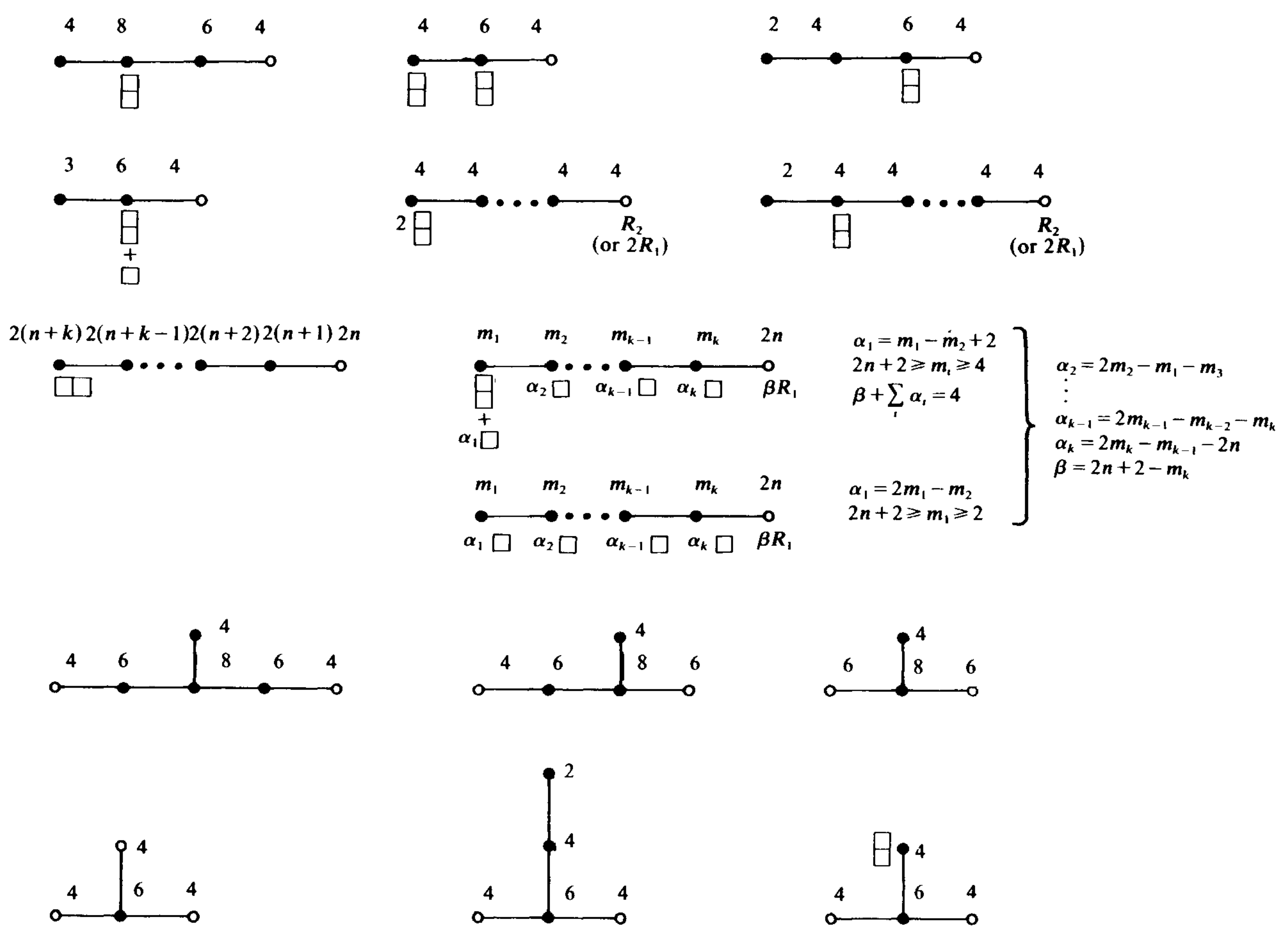


Table 1. (continued)



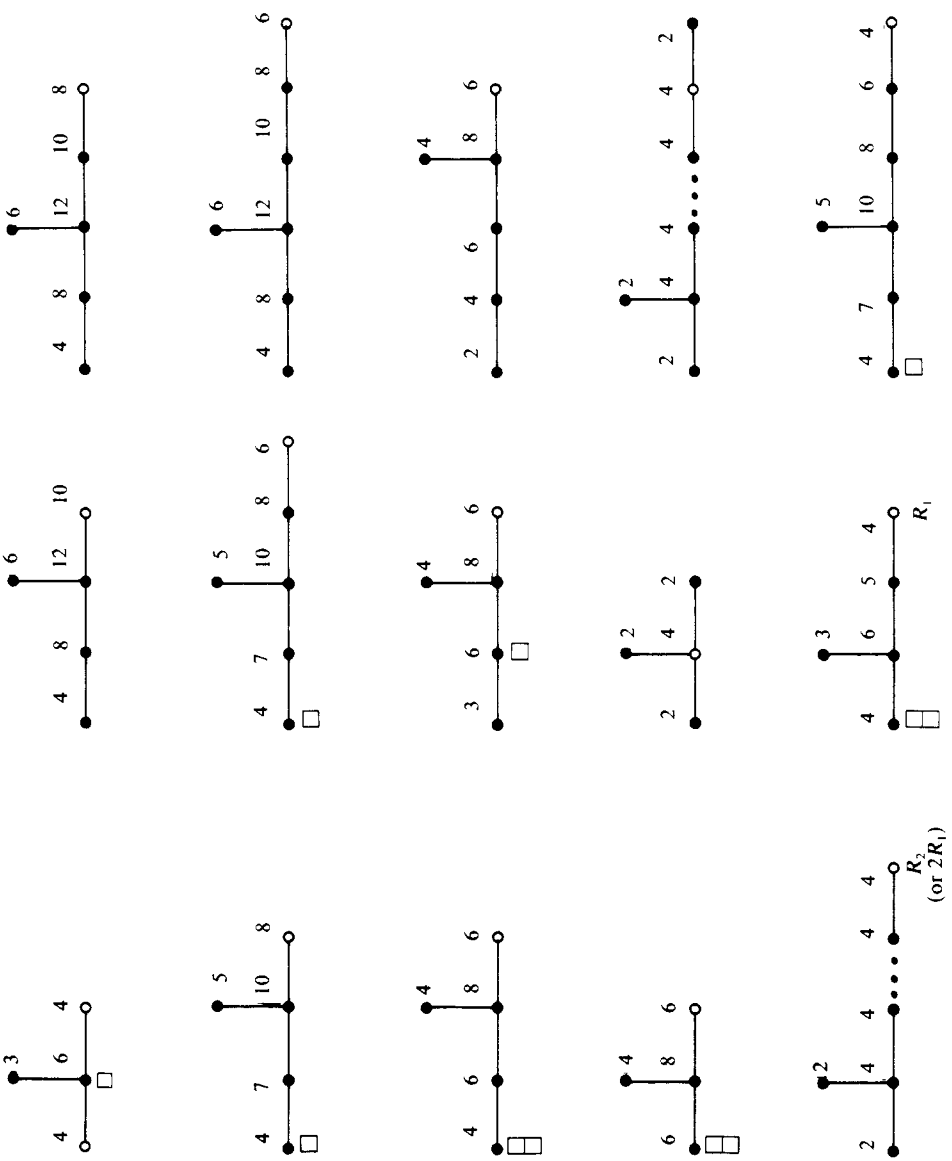
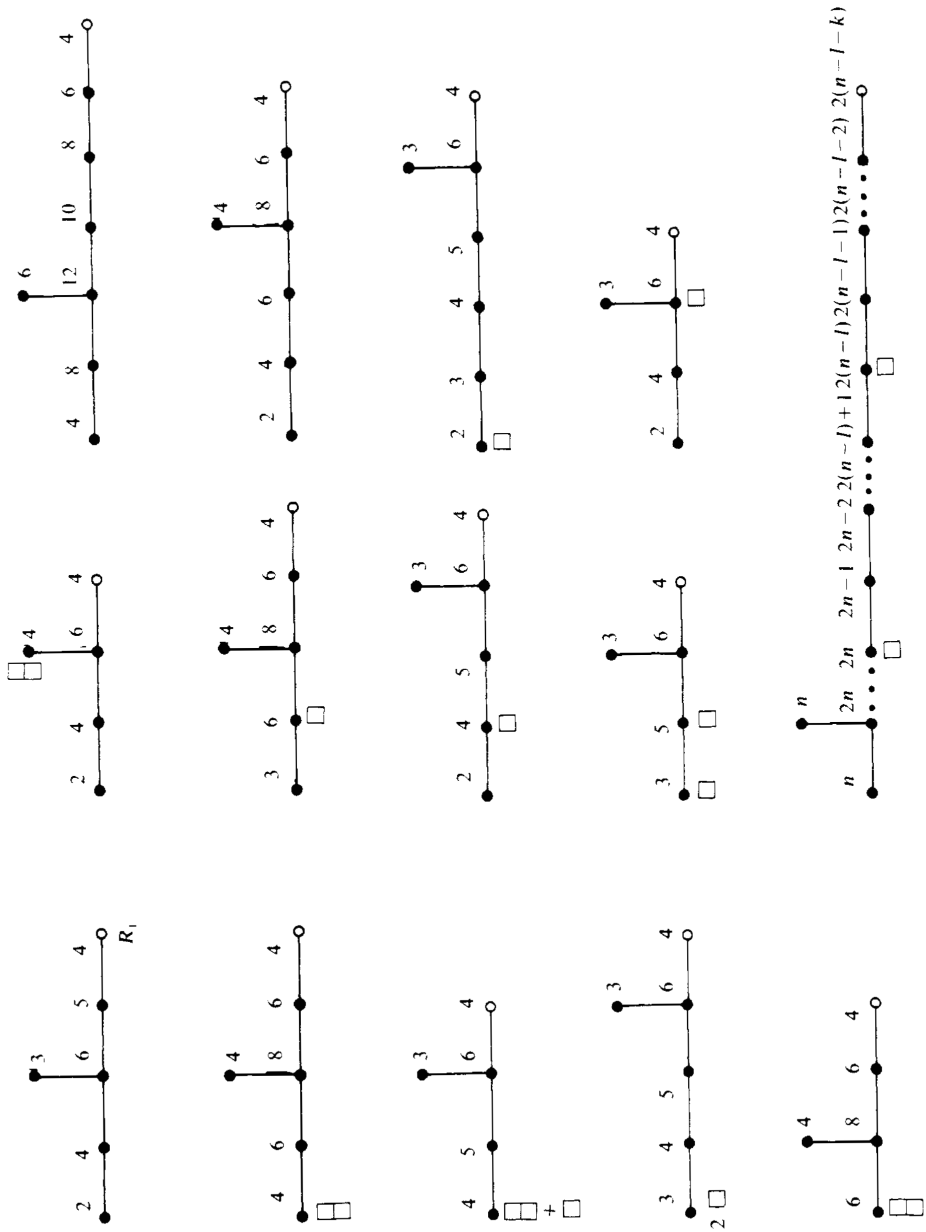

R1
R2
(or 2R1)

Table 1. (continued)



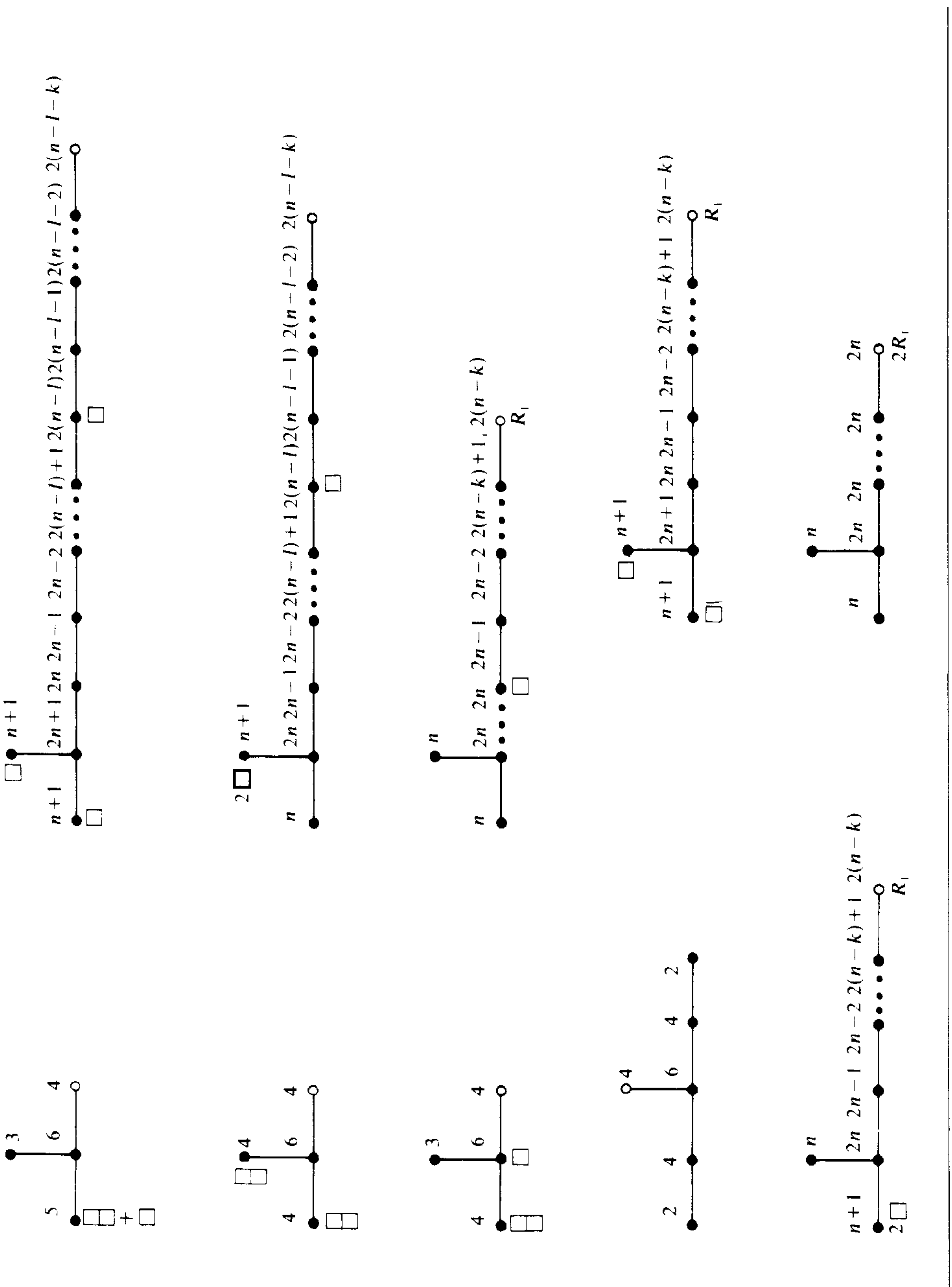
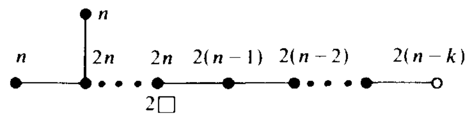
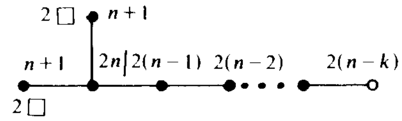
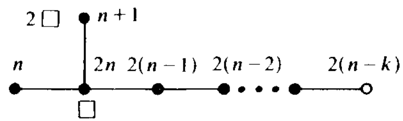
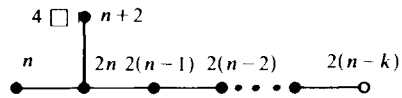


Table 1. (continued)









C All diagrams in table 1 of Jiang and Zhou (1984b)

† If the dots with value 6 have ⊟ (three boxes in a column), they always may have ⊟ (two boxes in a column) + 2□, or 6□. For simplicity we only list ⊟ (three boxes in a column) in diagrams. Similarly if the dots with value m have ⊟ (two boxes in a column), instead ⊟ (two boxes in a column) they may have $(m-2)$□.

Table 2. The dimension and Dynkin index of classical groups representations.

Representation R_t		Dimension	$T(R_t)$	Range of m
SU(m):	$\square$	m	$\frac{1}{2}$	$2 \leqslant m$
	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\frac{1}{2}m(m-1)$	$\frac{1}{2}(m-2)$	$4 \leqslant m$
	$\square\square$	$\frac{1}{2}m(m+1)$	$\frac{1}{2}(m+2)$	$3 \leqslant m$
	$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$	$\frac{1}{6}m(m-1)(m-2)$	$\frac{1}{4}(m-2)(m-3)$	$6 \leqslant m \leqslant 8$
	$m-1\left\{\begin{smallmatrix}\square\square\\\square\\\vdots\\\square\end{smallmatrix}\right.$	m^2-1	m	$2 \leqslant m$
SO($2m+1$)	R_f	$2m+1$	1	$3 \leqslant m$
	R_s	2^m	2^{m-3}	$3 \leqslant m \leqslant 6$
	R_a	$m(2m+1)$	$2m-1$	$3 \leqslant m$
SO($2m$)	R_f	$2m$	1	$4 \leqslant m$
	R_s	2^{m-1}	2^{m-4}	$4 \leqslant m \leqslant 7$
	R_a	$m(2m-1)$	$2m-2$	$4 \leqslant m$
Sp($2m$)	R_1	$2m$	1	$2 \leqslant m$
	R_2	$m(2m-1)-1$	$2m-2$	$2 \leqslant m$
	R_a	$m(2m+1)$	$2m+2$	$2 \leqslant m$

R_f, R_s and R_a refer to the fundamental representation, spinor representation and adjoint representation, respectively.

Acknowledgments

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